PARAMETER INSENSITIVE MATCHING CIRCUITS FOR LOW COST INTEGRATED CIRCUITS*

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ABSTRACT

A new theory for increasing the tolerance of matching networks to load variations is presented. This theory is based on matching the angles of the changes in $S_{11}(f)$ due to frequency and load variations, and its use can double allowed device tolerances in many cases. As a byproduct, this theory also shows why Chebyshev matching filters have poor tolerances. Examples are given showing as much as an order of magnitude improvement in circuit tolerances.

INTRODUCTION

The manufacture of microwave amplifiers, mixers and oscillators must include component and device tolerances in the design procedure if lower cost and more reliable circuits are to be produced. A new theory for maximizing the tolerance of matching networks to load variations is presented here. The tolerance of $S_{11}(f)$ to variations in the load, or any other component, is maximized when the variations cause a change in $S_{11}(f)$ that is perpendicular to the direction of $S_{11}(f)$. Knowledge of the angles of the differential changes in $S_{11}(f)$ with frequency and load variations allow the design of matching networks with a maximum tolerance over a band of frequencies.

The concern here is to maximize the tolerance of a circuit to variations in the active device used. For the lossless networks discussed here, it has been found that the magnitude and frequency variation of the input reflection coefficient determines the sensitivity to load variations. Simple formulae are presented for both differential and large change sensitivities in terms of S-parameters. These formulae, along with the nature of matching networks allows us to establish worst case tolerances, and show how matching networks capable of doubling the typical worst case load tolerance may be designed. Also, the rapid changes with frequency due to the ripples of a Chebyshev network will be shown to decrease its tolerance relative to what is possible with matching networks using a flat mismatch equal to the peak of the Chebyshev. Simulations of narrowband and broadband feedback FET amplifiers illustrate these conclusions.

THEORY

By using an ABCD matrix representation of a matching network, we may see the transformation from the load immittance to the input reflection coefficient as two bilinear transformations. This means that since changes in the real and imaginary parts of the load are orthogonal, resulting changes in the input reflection coefficient will be orthogonal. Since we usually want to increase the tolerance of the circuit to reactive variations in the load, we may exploit this orthogonality to increase our tolerance. The following equation will prove useful, and is derived assuming $Z_L(f) = R + j\,X(f)$:

$$\frac{\partial S_{11}(f)}{\partial X(f)} = j\,\frac{S_{21}(f)\,S_{12}(f)}{2\,Z_L(f)}. \qquad (1)$$

Figure 1 shows the center of the reflection coefficient plane with a circle enscribing the maximum tolerable reflection, Γ_{max}. A point, $S_{11}(f_o)$ is located inside this circle and represents a point on the curve of input reflection coefficient versus frequency for the amplifier. Increases and decreases in the device reactance, typically the input capacitance, will cause the point $S_{11}(f_o)$ to move. This differential movement is given exactly by equation (1) for any network describable via S-parameters.

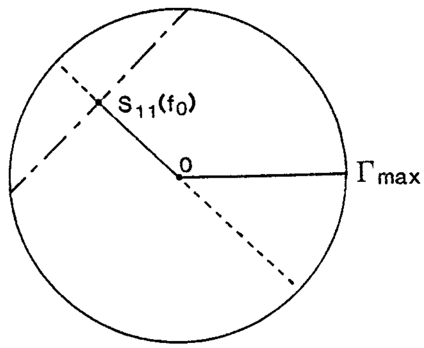

Figure 1. Reflection plane. The circle of maximum tolerable reflection, Γ_{max}, and the circuit reflection coefficient at f_o, $S_{11}(f_o)$ are shown. The dashed and alternating lines show the minimum and maximum absolute changes in $S_{11}(f_o)$, respectively.

The alternating line of Figure 1 shows the maximum symmetric tolerance about the point $S_{11}(f_o)$. The dashed line of Figure 1 shows an asymmetric tolerance which for practical purposes should be limited to its maximum symmetric variation. This shows how variations in the device capacity will have a minimum tolerance if they cause radial excursions in $S_{11}(f_o)$ and a maximum tolerance if they cause excursions of $S_{11}(f_o)$ along the alternating line of Figure 1. As a reminder, the tolerance in the device capacity would be that percentage change which caused $S_{11}(f_o)$ to lie on the Γ_{max} circle.

The next step toward a more tolerant network lies in the expression of the change in $S_{11}(f)$ with frequency as given by the equation

$$\frac{\partial S_{11}(f)}{\partial f} = j\,\frac{S_{21}(f)S_{12}(f)}{2Z_L(f)} \cdot \frac{\partial X}{\partial f} + \frac{S_{21}^2(f)}{2Z_L(f)} \cdot \left[\left(C\frac{\partial A}{\partial f} - A\frac{\partial C}{\partial f}\right) Z_L^2(f) + \left(C\frac{\partial B}{\partial f} + D\frac{\partial A}{\partial f} - A\frac{\partial D}{\partial f} - B\frac{\partial C}{\partial f}\right) Z_L(f) + D\frac{\partial B}{\partial f} - B\frac{\partial D}{\partial f}\right]. \qquad (2)$$

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where A, B, C, and D are the parameters of the ABCD matrix for the input matching network. Equation (2) shows that for all low Q or resistive matching networks the absolute direction of $\partial S_{11}(f)/\partial f$ is exactly the same as that of $\partial S_{11}(f)/\partial X$, depending only on the sign of $\partial X/\partial f$. For these networks reactive variations in the load have their maximum tolerance when the $S_{11}(f)$ is a constant over the passband, i.e. the matching filter has a flat mismatch response. While practical networks are not resistive and rarely low Q, equation (2) leads to the conclusion that matching networks should provide a flat mismatch at least over the center of the band and should not have large ripples such as a Chebyshev filter. These conclusions will be elaborated on via examples on Chebyshev and Butterworth matching filters, as well as a broadband FET feedback amplifier.

Another result of this study is given by equation (3) which gives the maximum percentage tolerance of a series or parallel load capacitance assuming that $S_{11}(f_o) = 0$, the matching network is lossless, the network is also reciprocal, and the percentage change in the capacitance is small:

$$\% \text{ tolerance} = \frac{200}{Q_f} \cdot \frac{\Gamma_{max}}{\sqrt{1-\Gamma_{max}^2}} , \qquad (3)$$

where Q_f is the Q of the load at f_o.

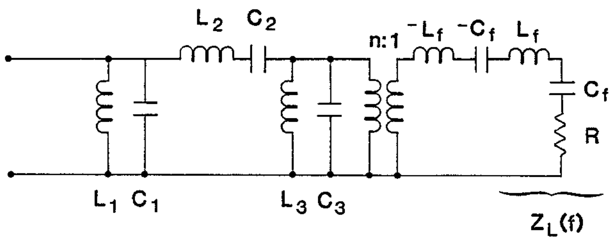

Figure 2. Idealized bandpass matching filter for testing tolerances to load, $Z_L(f)$ and circuit variations. See Table I and Figures 3 and 4.

RESULTS

The network of Figure 2 was used to examine the tolerance of ideal Chebyshev and Butterworth responses to variations in the load capacitance. The load is a narrowband model of the input impedance of a 250 μ wide FET used in the 3.7 to 4.2 GHz range. Note that ideal responses were obtained by neutralizing the load reactances with negative components and using an ideal transformer. Figures 3 and 4 are plots of third order Chebyshev and Butterworth bandpass filters with a normal and 2% high FET capacitance. These plots are only from 3.75 to 4 GHz for clarity, and have the normal capacitance, .3 pF, as the circled points and the high value as triangular points. These filters were designed for peak reflections less than .26 from 3.7 to 4.2 GHz.

The problem with the Chebyshev filter is not only that the reactance variation causes a change in $S_{11}(f)$ that is not quite tangent to $S_{11}(f)$ at the ripple peaks, but that the change in $S_{11}(f)$ due to reactive load variations becomes tangent to the curve of $S_{11}(f)$ at higher frequencies. This means that the rapid magnitude change in $S_{11}(f)$ causes reactive variations in in the load to cause near radial variations in $S_{11}(f)$ and thus a lower tolerance. Chebyshev responses are only optimal when passband and stopband responses are considered. As can be seen from Figures 3 and 4, the mismatched Butterworth response has a greater tolerance in the midband even though its smallest reflection is equal to the peak reflection of the Chebyshev. The Butterworth filter's tolerance is reduced at the band edges because variations in the load reactance cause changes in $S_{11}(f)$ which begin to be normal to the variation in $S_{11}(f)$ with frequency. Since the Butterworth filter has near ideal tolerance properties in its midband, modifying its band edge response will lead toward optimally load tolerant filters.

The subject of load tolerant filters in lossless reciprocal structures demands selecting filter properties that yield the highest tolerance to variatons in reactive or resistive variations in the load. Reciprocal structures must rely on loss or angular relations for tolerant design, as seen via the $S_{21}(f)$ and $S_{12}(f)$ terms in equations (1) and (2). The $S_{12}(f)$ term in equation (1) shows how circulators and isolators allow tolerance to load variations in nonreciprocal networks.

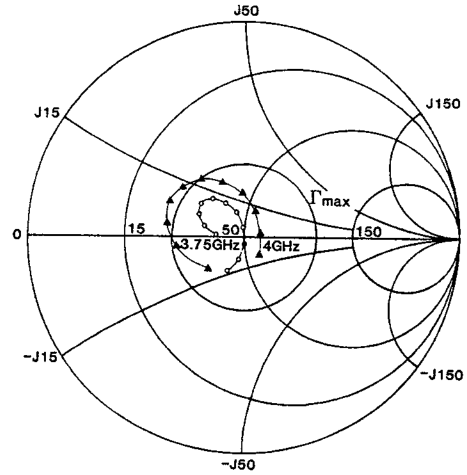

Figure 3. Plot of Chebyshev filter response (circles) and response with load capacitance, C_f, increased by 2% (triangles). Also shown is the maximum allowable reflection, Γ_{max} = .333. Only 3.75 to 4 GHz of the 3.7 to 4.2 GHz response is shown for clarity. Note response variation to load capacitance becomes tangent to the response curve as frequency is increased (dashed lines). See circuit in Figure 2.

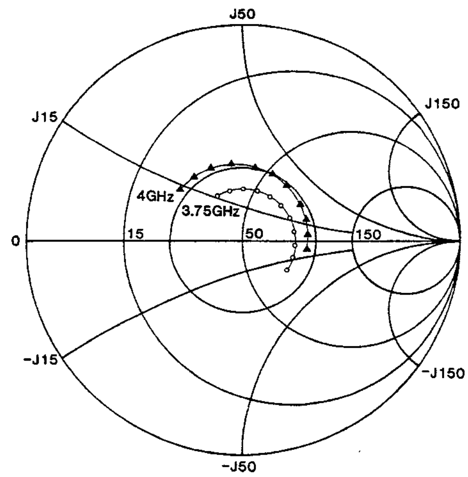

Figure 4. Plot of Butterworth filter response (circles) and response with load capacitance, C_f, increased by 2% (triangles). Also shown is the maximum allowable reflection, Γ_{max} = .333. Only 3.75 to 4 GHz of the 3.7 to 4.2 GHz response is shown for clarity. Note response variation to load capacitance becomes tangent to response curve in the midband (dashed lines). See circuit in Figure 2.

TABLE I

BANDPASS MATCHING FILTER TOLERANCES

Filter	Circuit Parameters: Q	Circuit Parameters: ρ max	Allowed Element Tolerances (%) for Γ_{max} = .333 and BW = 3.7-4.2 GHz (see Figure 2): Device C_f	Circuit 1	Circuit 2
Chebyshev	6.25	.278	1.2%	>.5% L_2	>.5% C_2
Butterworth	4	.28 (.25 midband)	.8%	>1% C_3	>1% L_3
Butterworth	4	.21 (.164 midband)	2.1%	>3% C_1	>4% L_2
Butterworth	.25	.278	2.2%	>6% n	>8% L_f
Ideal (eqtn 3)	0	0.0	3.9%		

TABLE II

MATCHING FILTER TOLERANCES

Broadband FET Amplifier (1-9 GHz)

Filter	Matching Filter C_1(pF)	Matching Filter L_2(nH)	Allowed Element Tolerances (%) for Γ_{max} = .333 and S_{21}>1.75: Device C_f	Circuit 1	Circuit 2
Tuned	.37	.92	>.5%	>.5% L_2	>1% g_m
Flattest	.2	.7	>6%	>11% g_m	>13% R_{FB}

A second point of clarification concerns the straight lines drawn in Figure 1. Of course, the bilinear property of networks causes all variations to have a finite curvature. This is most familiar to those using the MAP function of COMPACT. The result of this is to cause the calculations based on the straight lines of Figure 1 to be approximate.

Various bandpass networks represented by Figure 2 were tested for their tolerance characteristics over the 3.7 to 4.2 GHz band with Γ_{max} = .333. Table I provides information not only about load variations, but also about tolerances to filter components and the results of equation (3). The tolerance information is given for a maximum $S_{11}(f)$ = .333 with only one component being varied. The additional tolerances in Table I are for the first and second most sensitive network elements, as labeled, and their actual tolerance is less than the next higher integer.

Table II gives the tolerance results for a broadband FET feedback amplifier. This amplifier used a 750 μ wide FET of C_f = .6pF and g_m = .065 mhos with an LC matching filter (C_1, L_2) and a feedback resistor (R_{FB}) of 180 ohms. The FET model included source inductance and other pertinent components. Two different different matching filters were used with this amplifier, and have responses shown in Figure 5. The first design was "tuned" to provide zero reflection near 7 GHz, and results in a familiar humped response. The second response was designed to give a finite but flatter VSWR across the entire band. This latter design is less straightforward, but results in a much greater tolerance to both the FET and the circuit elements.

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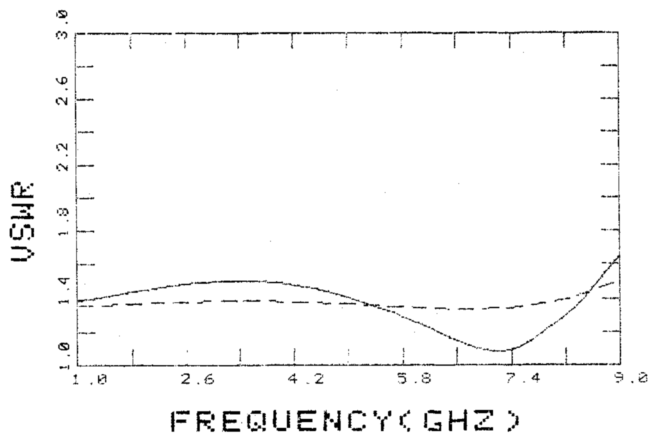

Figure 5. Input VSWR of 1-9 GHz FET feedback amplifier. Solid line is response with a tuned matching filter, ρ_{max} = .248. Dashed line is with a flat matching filter, ρ_{max} = .2.

CONCLUSION

In addition to the presentation of several equations useful for designing more tolerant circuits, it has been shown that the peaks and rapid magnitude changes of a Chebyshev filter create a low tolerance response. The use of matching filters which have a flat mismatch across most of the band was shown to provide a large improvement in device and circuit tolerances in all but the large reflection with high Q cases. High tolerance reciprocal networks are achieved by designing the circuit response so element variations cause response variations which are perpendicular to the response vector. The response should also be as flat as possible. The synthesis of maximally tolerant filters will aid the production of microwave circuits by simplifying the tuning procedure.